

Exercise 1

The following algebraic specification of integers is given:

```
sort Z;  
operations  
  zero : -> Z;  
  succ : Z -> Z;  
  pre  : Z -> Z;  
  add  : Z x Z -> Z;  
declare i, j : Z;  
axioms  
  pre(succ(i)) == i;  
  succ(pre(i)) == i;  
  add(zero,i) == i;  
  add(succ(i),j) == succ(add(i,j));  
  add(pre(i),j) == pre(add(i,j));
```

- Define the corresponding signature $\Sigma = \langle SN, FN, domN, ranN \rangle$.
- Define the corresponding word algebra $A = \langle S, F, dom, ran \rangle$.
- Define the relationship between the signature and the word algebra by a denotation $\delta : \Sigma \rightarrow A$.
- From the word algebra A , construct the quotient algebra QA . In which way are the elements of the quotient algebra defined? How are they related to integer numbers?
- Define a final algebra FA .
- Define a homomorphism $h : QA \rightarrow FA$.

Due date: 29.11.04